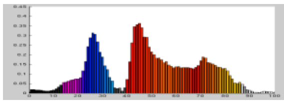


Dr. Qadri Hamarsheh

Signals and Systems
Lecture 1

Outline

- **Introduction- Systems and Signals.**
- **Application Areas.**
- **Fundamental Concepts:**
  - ✓ **Classifications of signals and systems.**
  - ✓ **Mathematical Models.**
- **Continuous-Time Signals (C-T Signals).**

Introduction- Systems and Signals

Signal: a time-varying voltage (or other quantity) that generally carries some information, in other words, signal is a function of the time variable t .

Signal Processing: the field of techniques used to extract information from signals.

Mathematical Model: we use mathematical language to describe the signals and systems (Electrical, Electronic, Economic, mechanical, etc.).

Firstly, we must design the mathematical model for any physical system.

Example: Radio Wave System: it takes in a radio frequency (RF) signal through the antenna and from it creates the sound.

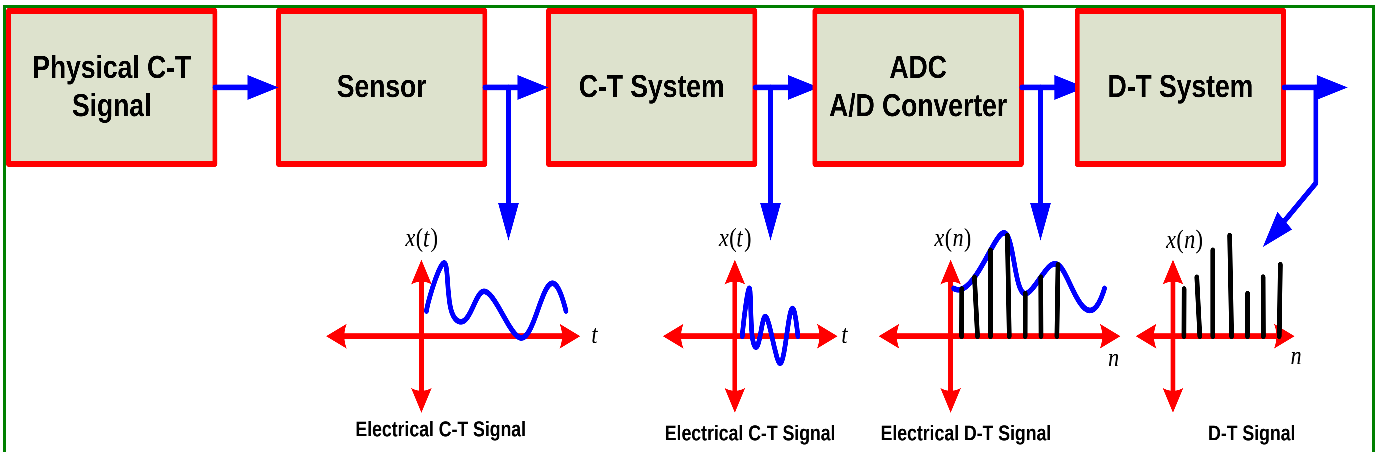
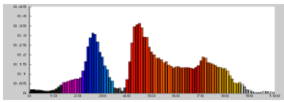


Figure 1:1 Systems Structure

Application Areas

- **Telecommunications**: modems, fax, cell phone.
- **Speech and Audio**: voice mail, audio compression.
- **Medical**: MRI (Magnetic Resonance Imaging), CT-scan.
- **Automotive**: Engine control.
- **Image Processing**: Compression, Enhancement, Restoration.
- **Radar, Sonar**.



Fundamental Concepts:

Classifications of signals and systems

First Classification:

- **Deterministic signals and systems.**
- **Random signals and systems.**

Second Classification:

- **Continuous signals and systems.**
- **Discrete signals and systems.**

Third Classification:

- **Analog signals and systems**: Continuous in time and amplitude.
- **Discrete signals and systems**: Continuous amplitude signal is defined for certain time instances, these signals can be obtained from analog signals by doing sampling (random or uniform sampling).
- **Digital Signals**: discrete in time and discrete in amplitude (using M-levels quantization to be used in computers), these signals can be obtained from analog signals by doing sampling process followed by quantization process (Binary signals are special case of the digital signal).

> Signal classification helps us to choose the most suitable type of analysis for a particular signal.

Mathematical Models

There are many similarities between C-T and D-T signals and systems, as shown in the table 1.

Table 1: Signals and Systems

C-T Mathematical Models	D-T Mathematical Models
Time Domain	
1. Differential Equations	1. Difference Equations
2. C-T Convolution (Integral)	2. D-T Convolution (Summation)
Frequency Domain	
3. C-T Fourier Analysis and Frequency Response	3. D-T Fourier Analysis and Frequency Response
4. C-T Laplace Transform and Transfer Function	4. D-T Z-Transform and Transfer Function

Continuous-Time Signals (C-T Signals)

Examples: voltage, current, audio signals such as speech or music waveform, bioelectric signals such as ECG. There are several elementary signals that occur prominently in the study of digital signals and digital signal processing.

Some Important C-T Signals:

1. **Step signal (Unit Step).**
2. **Ramp signal.**
3. **Unit pulse signal.**
4. **Periodic signals.**
5. **Rectangular pulse.**

1. Step signal (Unit Step)

This signal is used in DSP to represent a signal that switches on at a specified time and stays switched on indefinitely as shown in figure 1.2, the unit step function $u(t)$ is defined mathematically by:

$$u(t)=\begin{cases}1, & t\geq 0\\ 0, & t<0\end{cases}$$

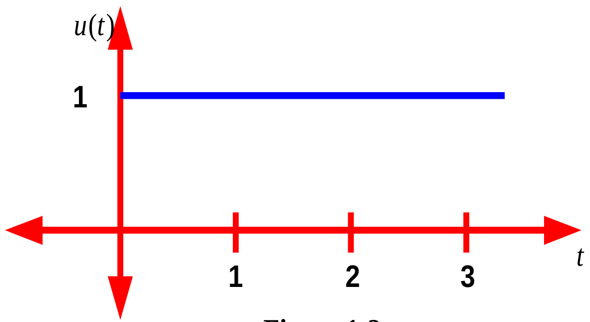


Figure 1-2

- The amplitude of $u(t)$ is equal to 1 for all $t \geq 0$.
- $ku(t)$ is the step function with amplitude k for $t \geq 0$, k-arbitrary nonzero real number.
- $x(t)u(t)$ is equal to $x(t)$ for $t \geq 0$ and is equal to zero for $t<0$. Thus multiplication of a signal $x(t)$ with $u(t)$ eliminates any nonzero values of $x(t)$ for $t<0$.
- Step function discontinuous at point $t=0$.

The unit step signal can model the act of switching on a DC source as shown in figure 1-3

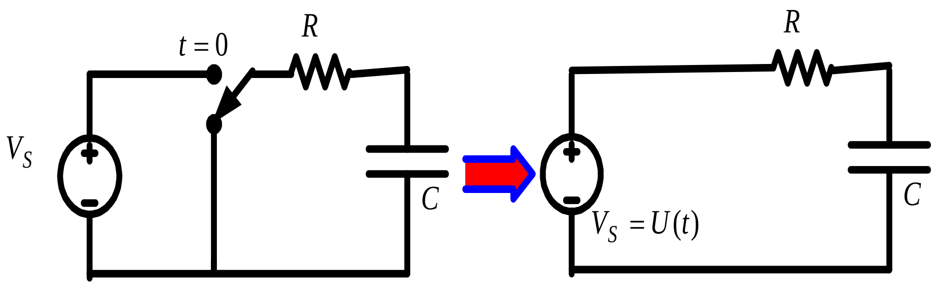


Figure 1-3 DC Source

The unit step signal is often used as a test signal to find the system response characteristics called Step Response.

2. Ramp signal

The unit-ramp signal $r(t)$ is defined mathematically by:

$$r(t)=\begin{cases}t, & t\geq 0\\ 0, & t<0\end{cases}$$

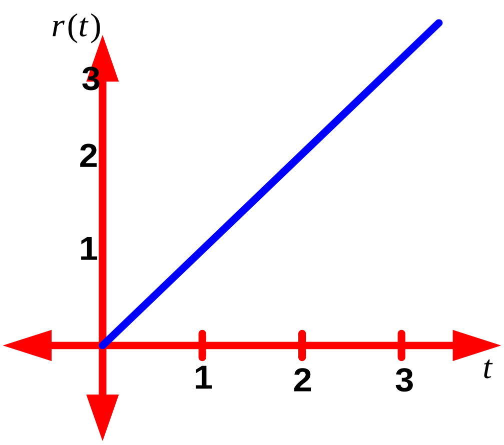
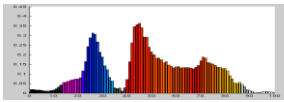


Figure 1-4

- For $t \geq 0$, the slope of $r(t)=1$, Thus the $r(t)$ has "unit slope".
- The ramp signal $kr(t)$ has slope k for $t \geq 0$, k- arbitrary nonzero scalar (real number).



- **Relationship between $u(t)$ and $r(t)$:**

   ▶ What is the $\int_{-\infty}^{t} u(\lambda)d\lambda$?

Algorithm:

- **Write a unit step as a function of λ.**
- **Integrate up to $\lambda = t$.**
- **Find the area as t changes.**

Implementation:

$$f(t) = \int_{-\infty}^{t} u(\lambda)d\lambda = 1 \cdot t = t = r(t) \Rightarrow r(t) = \int_{-\infty}^{t} u(\lambda)d\lambda$$

Running Integral of step signal equal to the ramp signal

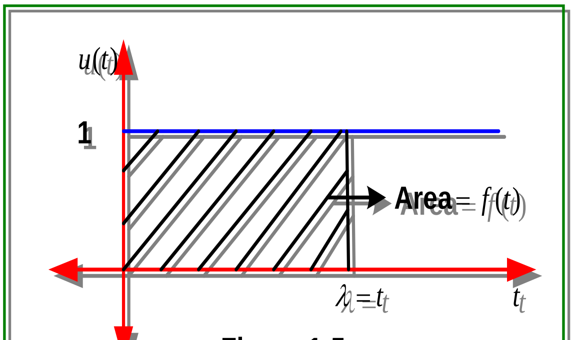


Figure 1-5

▶ What is $\dfrac{d\,r(t)}{dt}$?

- $\dfrac{d\,r(t)}{dt} = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$
- The first derivative of $r(t)$ with respect to t is equal to $u(t)$, except at $t = 0$, where derivative in not defined.

we can roughly say:

$$u(t) = \frac{d\,r(t)}{dt}$$

3. Unit Impulse $\delta(t)$ (Delta function or Dirac distribution)

One of the most important signals for understanding systems, the unit pulse defined as the following

$$\begin{cases} \delta(t) = 0, & t \neq 0 \\ \int_{-\varepsilon}^{\varepsilon} \delta(\lambda)d\lambda = 1, & for\ any\ real\ number\ \varepsilon > 0 \end{cases}$$

Or

$$\delta(t) = \begin{Bmatrix} +\infty, & t = 0 \\ 0, & t \neq 0 \end{Bmatrix} \quad and \quad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

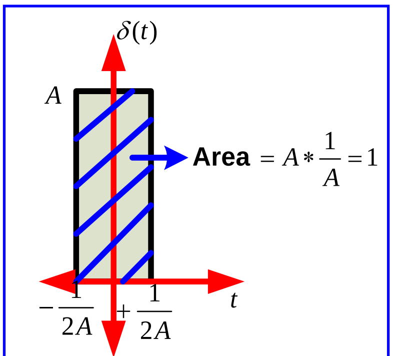


Figure 1-6

- $\delta(t)$ is zero for all nonzero values of t.
- The area under the impulse is 1, so $\delta(t)$ has a unit area.

- The pulse can be approximated by a pulse centered at the origin with amplitude A and a time duration $1/A$, where A is a very large positive number.
- $k\cdot\delta(t)$ is the impulse with area k:

$$\begin{cases} k\cdot\delta(t)=0, & t\neq 0 \\ \int_{-\varepsilon}^{\varepsilon} k\cdot\delta(\lambda)d\lambda = k, & \textit{for any real number } \varepsilon>0 \end{cases}$$

- The graphical interpretation of $\delta(t)$ is shown in figure 1-6.
- **Relationship between $\delta(t)$ and $u(t)$:**
  - The unit step is equal to the integral of the unit pulse $\delta(t)$:

$$u(t)=\int_{-\infty}^{t}\delta(\lambda)d\lambda\ ,\ \ \textit{all } t \textit{ except } t=0$$

  - The unit impulse is equal to the first derivative of the unit step:

$$\frac{d\,u(t)}{dt}=\delta(t)$$

- The unit impulse does not exist in practice.
- **Unit impulse different views**:
  - ✓ It is a pulse with infinite height, zero width and unit area.
  - ✓ $\delta(t)$ is not an ordinary function, it is defined in terms of its behavior inside an integral.
- The unit impulse is **used as a measure**: $\int_{-\infty}^{\infty} f(t)\cdot\delta(t)dt = f(0)$, for all functions f

4. Periodic signals

Periodic signal is a signal that repeats its values in regular intervals of periods; the most important examples are the trigonometric functions,
A continuous-time signal $x(t)$ is a periodic signal when:

$$x(t+T)=x(t) \text{ for all } t,\ -\infty<t<\infty,$$

it is also periodic with qT , q any positive integer.

Fundamental period: the smallest positive number T for which the above equation holds.

Examples of periodic signals are shown in figure 1-7.

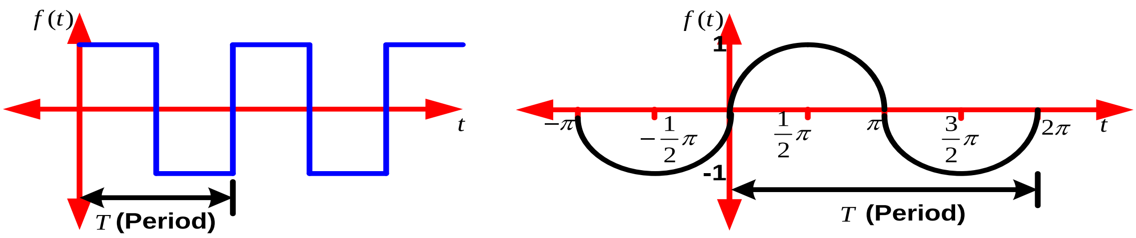
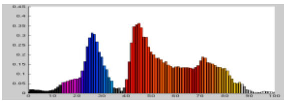


Figure 1-7

Sine function is periodic with period 2π: $\sin(t+2\pi)=\sin(t)$.



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Sinusoids: $x(t) = A\cos(\omega t + \theta), \quad -\infty < t < \infty$,

A-amplitude, ω-frequency in radian per second (rad/sec), θ-phase in radians, when the phase is non-zero, the entire wave form appears to be shifted in time by the amount $\frac{\theta}{\omega}$ seconds, $f = \frac{\omega}{2\pi}$ - frequency in hertz (cycles per seconds)

Sinusoids are periodic with period 2π:

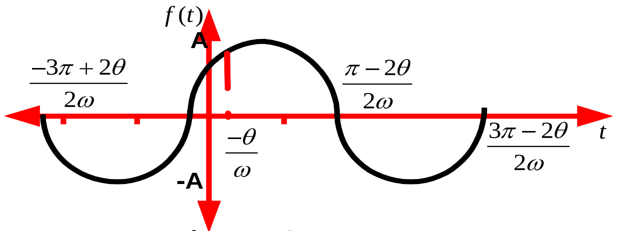

Figure 1-8

$$A\cos(\omega(t+\frac{2\pi}{\omega})+\theta) = A\cos(\omega t + 2\pi + \theta) = A\cos(\omega t + \theta),$$

thus the sinusoid is periodic with period $T = \frac{2\pi}{\omega}$

Sum of two periodic signals:

Suppose $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental periods T_1, T_2. Then the sum $x_1(t) + x_2(t)$ is periodic: $x_1(t+T_1) + x_2(t+T_2) = x_1(t) + x_2(t)$ for all t.

If the ratio $\frac{T_1}{T_2}$ can be written as the ratio $\frac{q}{r}$ of two integers q and r

$$\frac{T_1}{T_2} = \frac{q}{r} \Rightarrow r \cdot T_1 = q \cdot T_2$$

Then $x_1(t)$ and $x_2(t)$ are periodic with $r \cdot T_1$

Example: Suppose $x_1(t) = \cos(\frac{\pi t}{2})$ and $x_2(t) = \cos(\frac{\pi t}{3})$, then $x_1(t)$ and $x_2(t)$ are periodic with $T_1 = 4, T_2 = 6 \Rightarrow \frac{T_1}{T_2} = \frac{4}{6}; q = 2, r = 3$, the period of $x_1(t) + x_2(t) = r \cdot T_1 = 3 \cdot 4 = 12$ seconds.

5. Rectangular pulse function

Definitions:

$$1.\ \ \rho_\tau(t) = \begin{cases} 1 & \frac{-\tau}{2} \le t < \frac{\tau}{2} \\ 0 & t < \frac{-\tau}{2}, t \ge \frac{\tau}{2} \end{cases},$$

τ: fixed positive number equal to the time duration of the pulse.

$$2.\ \ \rho_\tau(t) = u(t+\frac{\tau}{2}) - u(t-\frac{\tau}{2})$$

- **Other important continuous signals**:

Triangular pulse, Exponential, Sawtooth, ….